Coimisiún na Scrúduithe Stáit

State Examinations Commission

Scéimeanna Marcála	Scrúduithe Ardteistiméireachta, 2007
Matamaitic Fheidhmeach	**Ardleibhéal**

Marking Scheme	Leaving Certificate Examination, 2007
Applied Mathematics	**Higher Level**

Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE APPLIED MATHEMATICS

HIGHER LEVEL

MARKING SCHEME

M32

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General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2).

2 The marking scheme shows one or more approaches to solving each question. In many cases there are other equally valid methods of solution.

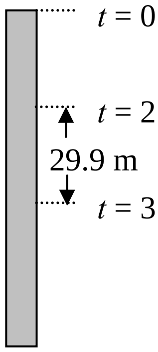
1. **(a)** A particle is projected vertically downwards from the top of a tower with speed u m/s. It takes the particle 4 seconds to reach the bottom of the tower.

During the third second of its motion the particle travels 29.9 metres.

Find

(i) the value of u

(ii) the height of the tower.



(i) $$s = ut + \tfrac{1}{2} ft^2$$

$$h = u(2) + 4.9(4)$$ 5

$$h + 29.9 = u(3) + 4.9(9)$$ 5

$$29.9 = u + 24.5$$

$$u = 5.4 \text{ m/s}$$ 5

(ii) $$s = ut + \tfrac{1}{2} ft^2$$

$$= 5.4(4) + 4.9(16)$$

$$= 100 \text{ m}$$ 5 20

OR

(i) Third second :

$$s = ut + \tfrac{1}{2} ft^2$$

$$29.9 = u(1) + 4.9(1)$$

$$u = 25$$ 5

First two seconds :

$$v = u + at$$

$$25 = u + 9.8(2)$$ 5

$$u = 5.4 \text{ m/s}$$ 5

1 **(b)** A train accelerates uniformly from rest to a speed v m/s.

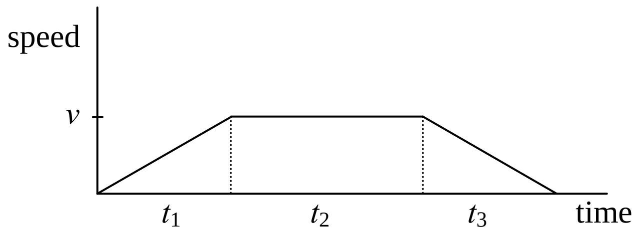
It continues at this speed for a period of time and then decelerates uniformly to rest.

In travelling a total distance d metres the train accelerates through a distance pd metres and decelerates through a distance qd metres, where $p < 1$ and $q < 1$.

(i) Draw a speed-time graph for the motion of the train.

(ii) If the average speed of the train for the whole journey is $\dfrac{v}{p+q+b}$, find the value of b.

(i)



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(*ii*)

$\frac{1}{2}t_1 v = pd$ 5

$t_2 v = d - pd - qd$ 5

$\frac{1}{2}t_3 v = qd$ 5

$$\text{Average speed} = \frac{d}{t_1 + t_2 + t_3}$$

$$= \frac{d}{\dfrac{2pd}{v} + \dfrac{d - pd - qd}{v} + \dfrac{2qd}{v}}$$ 5

$$= \frac{v}{p+q+1}$$ 5

$\Rightarrow \quad b = 1$

30

2. **(a)** Ship B is travelling west at 24 km/h. Ship A is travelling north at 32 km/h.

At a certain instant ship B is 8 km north-east of ship A.

(i) Find the velocity of ship A relative to ship B.

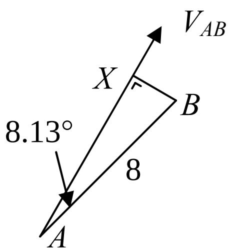
(ii) Calculate the length of time, to the nearest minute, for which the ships are less than or equal to 8 km apart.

(i)

$$\vec{V}_A = 0\vec{i} + 32\vec{j}$$
$$\vec{V}_B = -24\vec{i} + 0\vec{j}$$

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B \qquad 5$$
$$= 24\vec{i} + 32\vec{j} \qquad 5$$

$$\left.\begin{array}{ll}\text{magnitude:} & 40 \text{ km/h} \\ \text{direction:} & \text{East } 53.13° \text{ North}\end{array}\right\} \qquad 5$$



(ii)

$$\text{time} = \frac{2|AX|}{\vec{V}_{AB}} \qquad 5$$
$$= \frac{16\cos 8.13°}{40} \qquad 5$$
$$= 0.396 \text{ hours}$$
$$= 24 \text{ minutes} \qquad 5$$

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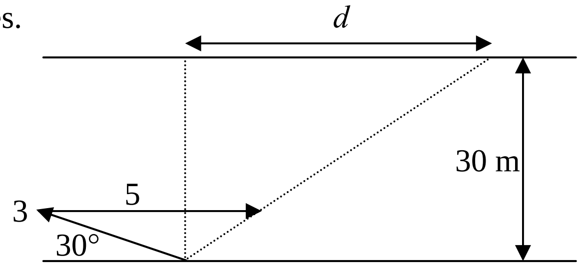
2 **(b)** A man can swim at 3 m/s in still water.
He swims across a river of width 30 metres.
He sets out at an angle of 30° to the bank.
The river flows with a constant speed of 5 m/s parallel to the straight banks.
In crossing the river he is carried downstream a distance *d* metres.



Find the value of *d* correct to two places of decimals.

$$\text{Time to cross} = \frac{30}{3\sin 30}$$

$$= 20 \text{ seconds}$$

$$d = (5 - 3\cos 30) \times 20$$

$$= (2.402) \times 20$$

$$= 48.04 \text{ metres}$$

5	
5	
5	
5	20

3. **(a)** A particle is projected with a speed of $7\sqrt{5}$ m/s at an angle α to the horizontal.

Find the two values of α that will give a range of 12.5 m.

$$r_{\vec{j}} = 0$$

$$7\sqrt{5}\sin\alpha.t - \tfrac{1}{2}gt^2 = 0 \qquad 5$$

$$\Rightarrow t = \frac{14\sqrt{5}\sin\alpha}{g} \qquad 5$$

$$\text{Range } = 7\sqrt{5}\cos\alpha.t$$

$$= 7\sqrt{5}\cos\alpha.\left(\frac{14\sqrt{5}\sin\alpha}{g}\right) \qquad 5$$

$$= 50\sin\alpha\cos\alpha$$

$$= 25\sin 2\alpha$$

$$\text{Range } = 12.5$$

$$25\sin 2\alpha = 12.5$$

$$\sin 2\alpha = \frac{1}{2} \qquad 5$$

$$\Rightarrow 2\alpha = 30^\circ \quad \text{or} \quad 150^\circ$$

$$\Rightarrow \alpha = 15^\circ \quad \text{or} \quad 75^\circ \qquad 5$$

25

3 **(b)** A plane is inclined at an angle 45° to the horizontal. A particle is projected up the plane with initial speed u at an angle θ to the **horizontal**.
The plane of projection is vertical and contains the line of greatest slope.

The particle is moving horizontally when it strikes the inclined plane.

Show that $\tan\theta = 2$.

$$r_j = 0$$

$$0 = u\sin(\theta - 45).t - \tfrac{1}{2} g\cos 45.t^2$$

$$\Rightarrow \quad t = \frac{2u\sin(\theta-45)}{g\cos 45}$$ 5

$$v_i = u\cos(\theta-45) - g\sin 45.t$$

$$= u\cos(\theta-45) - g\sin 45.\left(\frac{2u\sin(\theta-45)}{g\cos 45}\right)$$ 5

$$= u\cos(\theta-45) - 2u\sin(\theta-45)$$

$$v_j = u\sin(\theta-45) - g\cos 45.t$$

$$= u\sin(\theta-45) - g\cos 45.\left(\frac{2u\sin(\theta-45)}{g\cos 45}\right)$$ 5

$$= -u\sin(\theta-45)$$

$$\text{Landing angle} = 45° \Rightarrow \tan 45 = \frac{-v_j}{v_i}$$

$$1 = \frac{u\sin(\theta-45)}{u\cos(\theta-45) - 2u\sin(\theta-45)}$$

$$u\sin(\theta-45) = u\cos(\theta-45) - 2u\sin(\theta-45)$$

$$\tan(\theta-45) = \frac{1}{3}$$ 5

$$\frac{\tan\theta - 1}{1+\tan\theta} = \frac{1}{3}$$

$$3\tan\theta - 3 = 1 + \tan\theta$$

$$\Rightarrow \tan\theta = 2$$ 5

25

4. **(a)** A particle slides down a rough plane inclined at $45°$ to the horizontal. The coefficient of friction between the particle and the plane is $\frac{3}{4}$.
Find the time of descending a distance 4 metres from rest.

$$\left.\begin{aligned} R &= mg\cos 45 \\ mg\sin 45 - \mu R &= mf \end{aligned}\right\}$$ 5

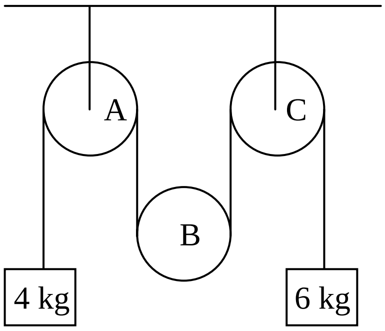
$$mg\sin 45 - \frac{3}{4}(mg\cos 45) = mf$$ 5

$$f = \frac{g}{4\sqrt{2}} \text{ m/s}^2$$ 5

$$s = ut + \tfrac{1}{2}ft^2$$
$$4 = 0 + \frac{1}{2}\left(\frac{g}{4\sqrt{2}}\right)t^2$$
$$t = \sqrt{\frac{32\sqrt{2}}{g}}$$
$$= 2.15\text{ s.}$$ 5

20

4 **(b)** A light inextensible string passes over a small fixed pulley A, under a small moveable pulley B, of mass *m* kg, and then over a second small fixed pulley C. A particle of mass 4 kg is attached to one end of the string and a particle of mass 6 kg is attached to the other end.
The system is released from rest.

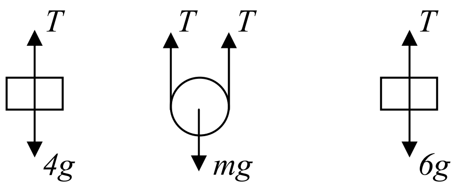


(i) On separate diagrams show the forces acting on each particle and on the moveable pulley B.

(ii) Find, in terms of *m*, the tension in the string.

(iii) If *m* = 9.6 kg prove that pulley B will remain at rest while the two particles are in motion.

(*i*)



5,5

(*ii*)

$$\left.\begin{aligned} T-4g &= 4p \\ T-6g &= 6q \end{aligned}\right\}$$ 5

$$mg-2T = m\left\{\frac{1}{2}(p+q)\right\}$$ 5

$$= \frac{m}{2}\left\{\left(\frac{T}{4}-g\right)+\left(\frac{T}{6}-g\right)\right\}$$

$$\Rightarrow T = \frac{48mg}{5m+48}$$ 5

(*iii*) $m=9.6 \;\Rightarrow\; \mathrm{T} = 47.04$ or $4.8g$

acceleration of 4 kg mass $= p = \dfrac{T}{4}-g = 0.2g \neq 0$

acceleration of 6 kg mass $= q = \dfrac{T}{6}-g = -0.2g \neq 0$

acceleration of pulley B $= \dfrac{1}{2}(p+q) = 0$ 5

$\Rightarrow$ pulley B will remain at rest while the two particles are in motion.

30

5. **(a)** A smooth sphere P, of mass 2 kg, moving with speed 9 m/s collides directly with a smooth sphere Q, of mass 3 kg, moving in the same direction with speed 4 m/s.
The coefficient of restitution between the spheres is e.

(i) Find, in terms of e, the speed of each sphere after the collision.

(ii) Show that the magnitude of the momentum transferred from one sphere to the other is $6(1+e)$.

(*i*) PCM $\quad 2(9) + 3(4) = 2v_1 + 3v_2$ 5

NEL $\quad v_1 - v_2 = -e(9-4)$ 5

$$\left.\begin{aligned} v_1 &= \frac{30-15e}{5} \quad \text{or} \quad 6-3e \\ v_2 &= \frac{30+10e}{5} \quad \text{or} \quad 6+2e \end{aligned}\right\}$$ 5

(*ii*)

$$\begin{aligned} \text{Impulse} &= 2(9) - 2(6-3e) \\ &= 6+6e \\ &= 6(1+e) \end{aligned}$$ 5

OR

$$\begin{aligned} \text{Impulse} &= 3(4) - 3(6+2e) \\ &= -6-6e \\ &= -6(1+e) \end{aligned}$$

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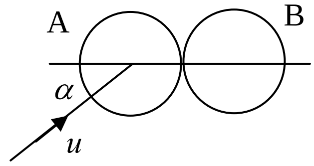
5 **(*b*)** A smooth sphere A, of mass 4 kg, moving with speed u, collides with a stationary smooth sphere B of mass 8 kg.
The direction of motion of A, before impact, makes an angle α with the line of centres at impact.



The coefficient of restitution between the spheres is $\frac{1}{2}$.

Find in terms of u and α

- **(i)** the speed of each sphere after the collision
- **(ii)** the angle through which the 4 kg sphere is deflected as a result of the collision
- **(iii)** the loss in kinetic energy due to the collision.

(i) PCM $\quad 4(u\cos\alpha) + 8(0) = 4v_1 + 8v_2$	5	
NEL $\quad v_1 - v_2 = -\frac{1}{2}(u\cos\alpha - 0)$	5	
$\Rightarrow \quad v_1 = 0 \quad$ and $\quad v_2 = \frac{1}{2}u\cos\alpha$		
Speed of A $= u\sin\alpha$ Speed of B $= \frac{1}{2}u\cos\alpha$	5	
(ii) Angle $= 90 - \alpha$	5	
(iii) KE before $= \frac{1}{2}(4)u^2 = 2u^2$ KE after $= \frac{1}{2}(4)\{u\sin\alpha\}^2 + \frac{1}{2}(8)\left\{\frac{1}{2}u\cos\alpha\right\}^2$ $= 2u^2\sin^2\alpha + u^2\cos^2\alpha$		
Loss in KE $= 2u^2 - 2u^2\sin^2\alpha - u^2\cos^2\alpha$	5	
$= 2u^2(1-\sin^2\alpha) - u^2\cos^2\alpha$ $= u^2\cos^2\alpha$	5	30

6. **(a)** A particle of mass m kg is suspended from a fixed point p by a light elastic string.
The extension of the string is d when the particle is in equilibrium.
The particle is then displaced vertically from the equilibrium position a distance not greater than d and is then released from rest.

(i) Show that the motion of the particle is simple harmonic.

(ii) Find, in terms of d, the period of the motion.

(i) Equilibrium position :

$$T_0 = kd \quad \Rightarrow \quad mg = kd \qquad 5$$

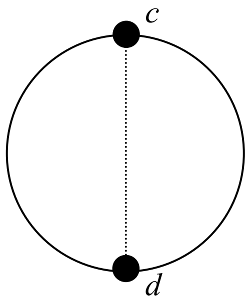
Displaced position :

$$\text{Force in dirn. of } x \text{ inc.} = mg - k(d+x)$$
$$= mg - kd - kx$$
$$= -kx \qquad 5$$
$$\text{Acceleration} = -\frac{kx}{m}$$
$$\Rightarrow \text{S.H.M. about } x = 0 \text{ with } \omega = \sqrt{\frac{k}{m}} \qquad 5$$

(ii)

$$\text{Period} = \frac{2\pi}{\omega}$$
$$= 2\pi\sqrt{\frac{m}{k}} \qquad 5$$
$$= 2\pi\sqrt{\frac{d}{g}} \qquad 5 \qquad 25$$

6 **(b)** A bead slides on a smooth fixed circular hoop, of radius r, in a vertical plane.
The bead is projected with speed $\sqrt{10gr}$ from the highest point c.
It impinges upon and coalesces with another bead of equal mass at d.
cd is the vertical diameter of the hoop.



Show that the combined mass will not reach the point c in the subsequent motion.

Let v be the speed of c : when it reaches d

Total energy at c = Total energy at d

$$\frac{1}{2}m(10gr) + mg(2r) = \frac{1}{2}mv^2 + mg(0) \qquad 5$$

$$v^2 = 14gr \qquad 5$$

Let v_1 be the speed of : the combined mass at d

$$mv + m(0) = 2mv_1$$

$$v_1 = \frac{1}{2}v \qquad 5$$

For the combined mass : to reach c with speed v_2

$$\frac{1}{2}(2m)(v_1)^2 + (2m)g(0) = \frac{1}{2}(2m)(v_2)^2 + (2m)g(2r)$$

$$m\left(\frac{v^2}{4}\right) = m(v_2)^2 + 4mgr$$

$$\frac{14gr}{4} = (v_2)^2 + 4gr$$

$$\Rightarrow (v_2)^2 = -\frac{1}{2}gr \qquad 5$$

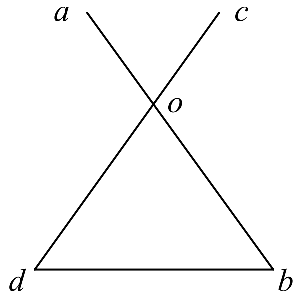
This is not possible $\Rightarrow$ the combined mass will not reach c. 5

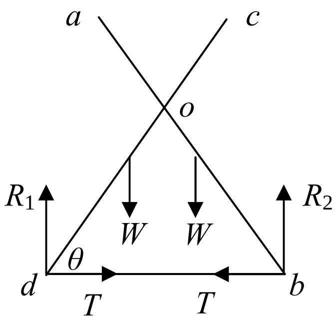
25

7. **(a)** *aob* and *cod* are two uniform rods, each of weight W, freely hinged at o.
$|ao| = |co| = 2\ell$ and $|ob| = |od| = 5\ell$.
The rods are in equilibrium in a vertical plane.

The ends b and d rest on a smooth horizontal plane and are connected by a light inextensible string of length 5ℓ.

Find the tension in the string.





$$R_1 + R_2 = 2W$$ 5

$$\theta = 60°$$

Take moments about d for system :

$$W\left(\frac{7\ell}{2}\right)\cos 60 + W\left(5\ell + \frac{3\ell}{2}\right)\cos 60 = R_2(5\ell)$$ 5

$$R_2 = W \quad \text{and} \quad R_1 = W$$ 5

Take moments about o for od :

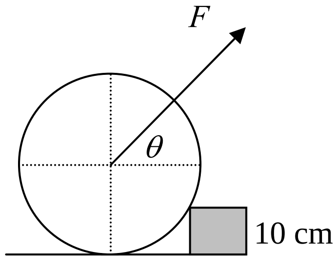
$$W\left(\frac{3\ell}{2}\cos 60\right) + T(5\ell \sin 60) = R_1(5\ell \cos 60)$$ 5

$$W\left(\frac{3\ell}{4}\right) + T\left(\frac{5\ell\sqrt{3}}{2}\right) = W\left(\frac{5\ell}{2}\right)$$

$$T = \frac{7W}{10\sqrt{3}} \quad \text{or} \quad 0.40W$$ 5 **25**

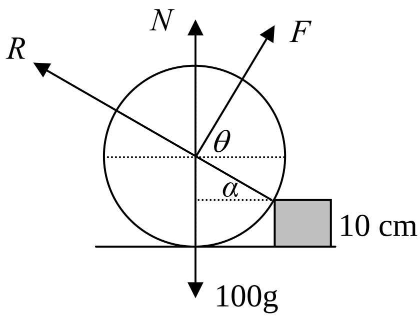
7 **(b)** A uniform disc of radius 25 cm and mass 100 kg rests in a vertical plane perpendicular to a kerb stone 10 cm high.

A force F is applied to the disc at an angle θ to the horizontal, where $\tan\theta = \frac{4}{3}$.



(i) Draw a diagram showing all the forces acting on the disc.

(ii) Find the least value of F required to raise the disc over the kerb stone.



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$$\sin\alpha = \frac{15}{25} = \frac{3}{5}$$ 5

horiz $$R\cos\alpha = F\cos\theta$$

$$R = \frac{3}{4}F$$ 5

vert $$R\sin\alpha + F\sin\theta + N = 100g$$ 5

$$N = 0$$

$$\left(\frac{3}{4}F\right)\left(\frac{3}{5}\right) + F\left(\frac{4}{5}\right) = 100g$$

$$F = 80g \quad \text{or} \quad 784\text{ N}$$ 5

25

8. **(a)** Prove that the moment of inertia of a uniform square lamina, of mass m and side $2r$, about an axis through its centre parallel to one of the sides is $\frac{1}{3}mr^2$.

$$\text{Let } \quad \text{M} = \text{mass per unit area}$$

$$\text{mass of element} = \text{M}\{2r\,\text{dx}\}$$

$$\text{moment of inertia of the element} = \text{M}\{2r\,\text{dx}\}\text{x}^2 \qquad 5$$

$$\text{moment of inertia of the lamina} = \text{M}\,2r\int_{-r}^{r}\text{x}^2\,\text{dx} \qquad 5$$

$$= \text{M}\,2r\left[\frac{\text{x}^3}{3}\right]_{-r}^{r} \qquad 5$$

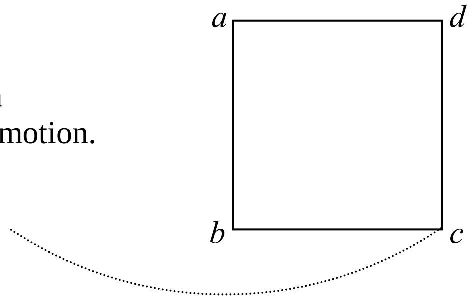
$$= \tfrac{4}{3}\text{M}\,r^4$$

$$= \tfrac{1}{3}\text{m}\,r^2 \qquad 5 \qquad 20$$

8 **(b)** **(i)** A uniform square lamina *abcd* of side $2r$ oscillates in its own plane about a horizontal axis through a , perpendicular to its plane.

If the period of small oscillations is $2\pi\sqrt{\dfrac{8}{3g}}$, find the value of r.

(ii) If the lamina is released from rest when ab is vertical, find the maximum velocity of corner c in the subsequent motion.



(i)

$$I = \frac{4}{3}(m)r^2 + \frac{4}{3}(m)r^2$$ 5

$$= \frac{8}{3}(m)r^2$$

$$Mgh = mgr\sqrt{2}$$ 5

$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$ 5

$$= 2\pi\sqrt{\frac{\frac{8}{3}(m)r^2}{mgr\sqrt{2}}}$$

$$= 2\pi\sqrt{\frac{8r}{3g\sqrt{2}}}$$

$$\Rightarrow r = \sqrt{2}$$ 5

(ii)

$$\text{Gain in KE} = \text{Loss in PE}$$

$$\tfrac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2}\left(\frac{8}{3}(m)r^2\right)\omega^2 = mg(r\sqrt{2} - r)$$ 5

$$\Rightarrow \omega = 1.467$$

$$\Rightarrow v = 4\omega = 5.87 \text{ m/s}$$ 5

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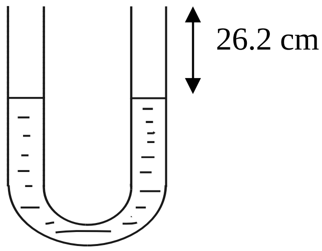
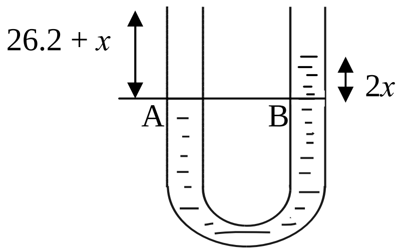
9. **(a)** A U-tube whose limbs are vertical and of equal length has mercury poured in until the level is 26.2 cm from the top in each limb.



Water is then poured into one limb until that limb is full.

The relative density of mercury is 13.6.

Find the length of the column of water added to the limb.



Let the length of column of water $= 26.2 + x$ 5

Pressure at A = Pressure at B

$1000\, g\, (26.2 + x)10^{-2} =$ 5

$13600\, g\, (2x)10^{-2}$ 5

$x = 1$ cm

Length of column of water $= 27.2$ cm. 5 20

OR

Let the length of column of water $= 0.262 + x$ 5

Pressure at A = Pressure at B

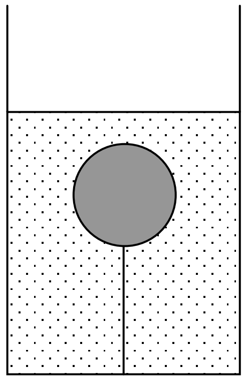
$1000\, g\, (0.262 + x) =$ 5

$13600\, g\, (2x)$ 5

$x = 0.01$ m or 1 cm

Length of column of water $= 27.2$ cm. 5

9 **(b)** A uniform solid sphere is held completely immersed in 500 cm^3 of water by means of a string tied to a point on the base of the container.
The tension in the string is 0.0784 N.



When 300 cm^3 of another liquid, of relative density 1.2 is added and thoroughly mixed with the water, the volume of the mixture is 800 cm^3 and the tension in the string is 0.1078 N.

Find

(i) the relative density of the mixture

(ii) the mass of the sphere.

(i) $\quad \text{mass of water} + \text{mass of liquid} = \text{mass of mixture}$

$$1000 \times 500 \times 10^{-6} + 1200 \times 300 \times 10^{-6} = \qquad 5$$

$$1000 \times s_m \times 800 \times 10^{-6} \qquad 5$$

$$s_m = \frac{43}{40} \quad \text{or} \quad 1.075 \qquad 5$$

(ii)

$$B = T + mg$$

water $\quad (1000Vg \text{ or}) \quad \dfrac{mg(1)}{s} = 0.0784 + mg \qquad 5$

mixture $\quad (1075Vg \text{ or}) \quad \dfrac{mg\left(\frac{43}{40}\right)}{s} = 0.1078 + mg \qquad 5$

$$(0.0784 + mg)\left(\frac{43}{40}\right) = 0.1078 + mg$$

$$m = 0.032 \text{ kg.} \qquad 5$$

30

10. **(a)** Solve the differential equation

$$\frac{dy}{dx} = y^2 \sin x$$

given that $y = 1$ when $x = \frac{\pi}{2}$.

$$\frac{dy}{dx} = y^2 \sin x$$

$$\int \frac{dy}{y^2} = \int \sin x \, dx \quad 5$$

$$-\frac{1}{y} = -\cos x + C \quad 5$$

$$y = 1, \ x = \frac{\pi}{2} \Rightarrow C = -1 \quad 5$$

$$\frac{1}{y} = \cos x + 1$$

$$\text{y} = \frac{1}{1 + \cos x} \quad 5$$

20

10 **(b)** The acceleration of a racing car at a speed of v m/s is

$$\left(1-\frac{v^2}{3200}\right) \text{ m/s}^2$$

The car starts from rest.

Calculate correct to two decimal places

(i) the speed of the car when it has travelled 1500 m from rest

(ii) the maximum speed of the car.

(*i*)

$$v\,\frac{dv}{dx} = \left(1-\frac{v^2}{3200}\right)$$

$$\int_0^v \frac{3200v}{3200-v^2}\,dv = \int_0^{1500} dx \qquad 5$$

$$\left[-1600\ln\left(3200-v^2\right)\right]_0^v = \left[x\right]_0^{1500} \qquad 5$$

$$1600\ln 3200 - 1600\ln\left(3200-v^2\right) = 1500$$

$$\frac{3200}{3200-v^2} = e^{15/16} \qquad 5$$

$$\Rightarrow v = 44.12 \text{ m/s} \qquad 5$$

(*ii*)

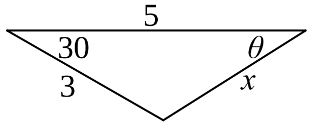
$$\text{acceleration} = 0$$

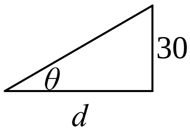
$$1-\frac{v^2}{3200} = 0 \qquad 5$$

$$\Rightarrow v = 56.57 \text{ m/s} \qquad 5 \qquad 30$$

Some Alternative Solutions

2 (b)





$$x^2 = 3^2 + 5^2 - 2(3)(5)\cos 30$$

$$x = 2.832$$ 5

$$\frac{\sin\theta}{3} = \frac{\sin 30}{2.832}$$

$$\theta = 31.985°$$ 5

$$\tan 31.985° = \frac{30}{d}$$ 5

$$d = 48.04 \text{ m}$$ 5 | 20

3 (b)

$$r_j = 0 \text{ on inclined plane}$$

$$0 = u\sin(\theta - 45).t - \tfrac{1}{2} g\cos 45.t^2$$

$$\Rightarrow \quad t = \frac{2u\sin(\theta-45)}{g\cos 45}$$ 5

Consider horizontal plane

$$v_j = 0$$

$$0 = u\sin\theta - gt$$ 5

$$\Rightarrow \quad t = \frac{u\sin\theta}{g}$$ 5

$$t = t$$

$$\frac{u\sin\theta}{g} = \frac{2u\sin(\theta-45)}{g\cos 45}$$

$$\sin\theta = 2\sqrt{2}\left\{\sin\theta\left(\frac{1}{\sqrt{2}}\right) - \cos\theta\left(\frac{1}{\sqrt{2}}\right)\right\}$$ 5

$$\sin\theta = 2\sin\theta - 2\cos\theta$$

$$\Rightarrow \tan\theta = 2$$ 5 | 25

3 (b)

$r_j = 0$ on inclined plane

$$u\sin(\theta - 45).t - \tfrac{1}{2} g\cos 45.t^2 = 0$$

$$\Rightarrow \quad t = \frac{2u\sin(\theta-45)}{g\cos 45} \qquad 5$$

$$= \frac{2u\{\sin\theta\cos 45 - \cos\theta\sin 45\}}{g\cos 45}$$

$$= \frac{2u}{g}\{\sin\theta - \cos\theta\}$$

$$\tan 45 = \frac{-v_j}{v_i}$$

$$\Rightarrow v_i = -v_j$$

$$u\cos(\theta-45) - g\sin 45.t = -u\sin(\theta-45) + g\cos 45.t \qquad 5$$

$$u\sin(\theta-45) + u\cos(\theta-45) = \sqrt{2}.g.t$$

$$u\{\sin\theta\cos 45 - \cos\theta\sin 45 + \cos\theta\cos 45 + \sin\theta\sin 45\} = \sqrt{2}.g.t \qquad 5$$

$$\sqrt{2}.u\sin\theta = \sqrt{2}.g.t$$

$$\sin\theta = \frac{gt}{u}$$

$$\sin\theta = \frac{g}{u}\left[\frac{2u}{g}\{\sin\theta - \cos\theta\}\right] \qquad 5$$

$$= 2\sin\theta - 2\cos\theta$$

$$2\cos\theta = \sin\theta$$

$$\Rightarrow \tan\theta = 2 \qquad 5 \qquad \boxed{25}$$

5 (b)

(iii) Take loss of kinetic energy in the $\vec{i}$ direction

KE before $= \dfrac{1}{2}(4)u^2\cos^2\alpha = 2u^2\cos^2\alpha$

KE after $= \dfrac{1}{2}(4)\{0\}^2 + \dfrac{1}{2}(8)\left\{\dfrac{1}{2}u\cos\alpha\right\}^2$

$= u^2\cos^2\alpha$

Loss in KE $= 2u^2\cos^2\alpha - u^2\cos^2\alpha$ 5

$= u^2\cos^2\alpha$ 5 | 10

6 (b)

Let v be the speed of c : when it reaches d

Total energy at c = Total energy at d 5

$$\frac{1}{2}m(10gr) + mg(2r) = \frac{1}{2}mv^2 + mg(0)$$ 5

$$v^2 = 14gr$$

Let v_1 be the speed of : the combined mass at d

$$mv + m(0) = 2mv_1$$ 5

$$v_1 = \frac{1}{2}v$$

At maximum height :

Gain in PE = Loss in KE

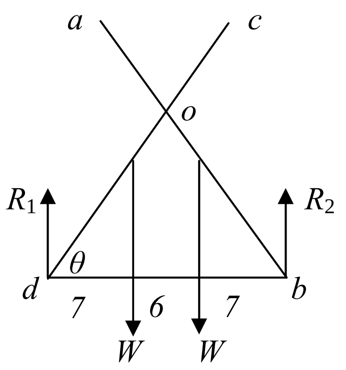
$$(2m)g(h) = \frac{1}{2}(2m)(v_1)^2$$

$$2gh = \frac{14gr}{4}$$ 5

$$h = \frac{7r}{4}$$ 5

As $h < 2r \Rightarrow$ the combined mass will not reach c. | 25

7 (a)



Lengths in ratio : $7x,\ 13x,\ 20x$ — 5

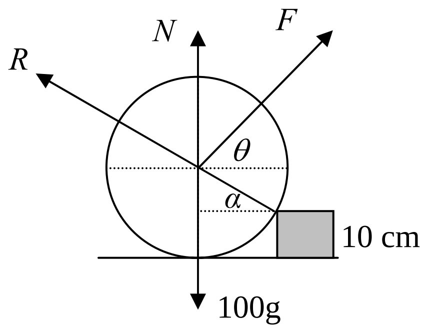
Take moments about b for system :

$$R_1(20x) = W(13x) + W(7x)$$ 5

$$R_1 = W$$ 5

[15]

7 (b)



5

$$F\cos\theta(0.15) + F\sin\theta(0.20) = 100g(0.20)$$ 5,5

$$F(0.6)(0.15) + F(0.8)(0.20) = 100g(0.20)$$ 5

$$F = 80g \quad \text{or} \quad 784\ \text{N}$$ 5

[25]

10(b)

(*ii*)

$$\frac{dv}{dt} = \left(1 - \frac{v^2}{3200}\right)$$

$$\int_0^v \frac{3200}{3200 - v^2}\, dv = \int_0^t dt$$

$$\left[20\sqrt{2}\ln\left(\frac{40\sqrt{2}+v}{40\sqrt{2}-v}\right)\right]_0^{\text{v}} = t$$

$$20\sqrt{2}\ln\left(\frac{40\sqrt{2}+v}{40\sqrt{2}-v}\right) - 0 = t$$

$$\frac{40\sqrt{2}+v}{40\sqrt{2}-v} = e^{t/20\sqrt{2}}$$

$$\frac{\left(40\sqrt{2}-v\right) - \left(40\sqrt{2}+v\right)(-1)}{\left(40\sqrt{2}-v\right)^2}\frac{dv}{dt} = \frac{1}{20\sqrt{2}}e^{t/20\sqrt{2}}$$

$$\frac{dv}{dt} = \frac{\left(40\sqrt{2}-v\right)^2}{80\sqrt{2}}\ \frac{1}{20\sqrt{2}}e^{t/20\sqrt{2}}$$

For maximum speed $\frac{dv}{dt} = 0$

$$0 = \frac{\left(40\sqrt{2}-v\right)^2}{80\sqrt{2}}\ \frac{1}{20\sqrt{2}}e^{t/20\sqrt{2}}$$ 5

$$\Rightarrow v = 40\sqrt{2}$$

$$\Rightarrow v = 56.57 \text{ m/s}$$ 5 | 10